DE BROGLIE HYPOTHESIS

Matter Waves

The dual nature of radiation prompted Louis de Broglie to extend it to material particles also. He reasoned that (i) nature is strikingly symmetric in many ways, (ii) our observable universe is composed entirely of radiation and matter, (iii) if light has a dual nature, perhaps matter has also. Since matter is composed of particles, his reasoning suggested that one should look for a wave-like behaviour for matter. In other words, de Broglie assumed that a wave is associated with a particle in motion, called *matter wave*, which may be regarded as localized with the particle. Again he suggested that the wavelength of matter wave be given by the same relationship, namely

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad (1)$$

where m is the mass and v is the velocity of the particle. This relation is often referred to as the *de Broglie relation*.

BOHRS QUANTIZATION RULE

An electron orbiting around a nucleus at a distance r is a bounded one and therefore the motion is represented by a standing wave. Only certain definite number of wavelengths can now exist in an orbit, otherwise the wave after

travelling once round the orbit will be out of phase with the previous one. Mathematically,

$$\oint \frac{ds}{\lambda} = n, \qquad n = 1, 2, \ldots \qquad (2)$$

where the integration is over one complete revolution. Substituting $\lambda = h/(mv)$, Eq. (2.2) reduces to

$$\oint mv\, ds = nh, \qquad n = 1, 2, \ldots \qquad (3)$$

which is a form of the *general quantization rule*. For circular orbits, $ds = r\, d\theta$

$$mvr\oint d\theta = nh \quad \text{or} \quad mvr = \frac{nh}{2\pi}, \qquad n = 1, 2, \ldots \qquad (4)$$

which is *Bohr's quantization rule*.

THE UNCERTAINITY PRINCIPLE

Heisenberg analyzed that no two canonically conjugate quantities can be measured Simultaneously. For the canonically conjugate variables x and px, mathematically,

The principle is stated as $\Delta x\, \Delta p_x \cong h$ (5

)The uncertainty relation can be illustrated by the *single-slit experiment*

Single-slit experiment. Consider a beam of monoenergetic electrons of speed v_0 moving along the y-axis. Let us try to measure the position x of an electron and its velocity component v_x in the vertical direction (x-axis). To measure x, we insert a screen S_1 which has a slit of width Δx (Figure). If an electron passes through this slit, its vertical position is known to this accuracy. This can be improved by making the slit narrower.

As the electron has a wave nature, it will undergo diffraction at the slit giving the pattern as in Figure . Just at the time of reaching the slit, the velocity v_x of the electron is zero. The formation of the diffraction pattern shows that the electron has developed velocity component v_x after crossing the slit. For the first minimum, the theory of diffraction gives

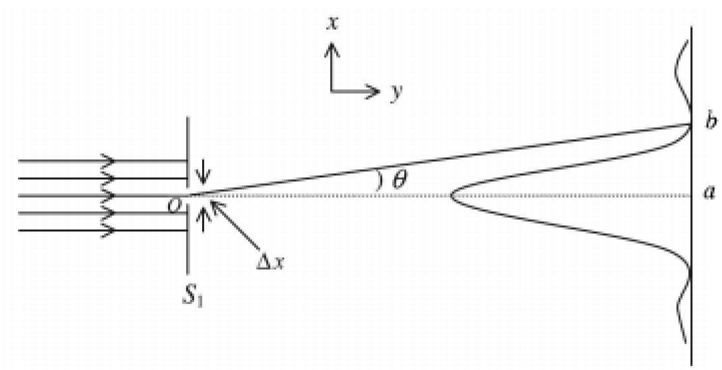

Figure Illustration of Heisenberg uncertainty principle—single-slit experiment.

$$\sin\theta \cong \theta = \frac{\lambda}{\Delta x}$$

where λ is the wavelength of the electron beam. Let t be the time of transit from o to a and v_{xb} be the value of v_x at b. Then

$$v_0 t = oa \qquad \text{and} \qquad v_{xb} = \frac{ab}{t}$$

$$\tan\theta \cong \theta = \frac{ab}{oa} = \frac{tv_{xb}}{v_0 t} = \frac{v_{xb}}{v_0}$$

we get

$$\frac{\lambda}{\Delta x} = \frac{v_{xb}}{v_0}$$

Replacing λ by h/mv_0 and taking v_{xb} as a rough measure of uncertainty Δv_x in v_x

$$\frac{h}{mv_0\Delta x} \equiv \frac{\Delta v_x}{v_0} \quad \text{or} \quad \Delta x \Delta p_x \equiv h$$

which is the desired relation. In the same way, we have

$$\Delta y \Delta p_y \equiv h \quad \text{and} \quad \Delta z \Delta p_z \equiv h$$

As the product of uncertainties is a universal constant, the more precisely we determine one variable, the less accurate is our determination of the other variable.

Before the introduction of the slit, the electrons travelling along the y-axis had the definite value of zero for p_x. By introducing the slit we measured the x-coordinate of the particles to an accuracy Δx, but this measurement introduced an uncertainty into the p_x values of the particles. Thus, the act of measurement introduced an uncontrollable disturbance in the system being measured which is a consequence of the wave particle duality.

Uncertainty Relations for other Variables

Uncertainty relations can also be obtained for other pairs of canonically conjugate variables. For a free particle moving along x-axis, energy E is given by

$$E = \frac{p_x^2}{2m} \quad \text{or} \quad \Delta E = \frac{p_x}{m}\Delta p_x = v_x \, \Delta p_x$$

Therefore,

$$\Delta E = \frac{\Delta x}{\Delta t}\Delta p_x \quad \text{or} \quad \Delta E \, \Delta t = \Delta x \, \Delta p_x$$

Hence

$$\Delta E \, \Delta t \equiv h$$

This equation indicates that if a system maintains a particular state for time Δt, its energy is uncertain at least by $\Delta E = h/\Delta t$.

The uncertainty for the pair of variables, component of angular momentum along the direction perpendicular to the plane of the orbit (L_z) of a particle and the angular position (ϕ) can be obtained as follows:

$$L_z^2 = 2IE$$

On differentiating, we get

$$L_z \, \Delta L_z = I\Delta E$$

where

$$L_z = I\omega = I\frac{\Delta \phi}{\Delta t}$$

Therefore,

$$I\,\frac{\Delta\phi}{\Delta t}\Delta L_z = I\,\Delta E$$

we have

$$\Delta\phi\,\Delta L_z = \Delta E\ \Delta t = h$$

These uncertainty relations are very useful in explaining number of observed phenomena which the classical physics failed.